Moody's Approach to Rating *i*th-to-Default Basket Credit-Linked Notes

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OVERVIEW

Credit derivatives permit the transfer of credit risk without the need to hold positions in the underlying instruments. The rapidly expanding credit derivatives market has created a large set of instruments for credit risk management. Moody's now rates many products associated with portfolios of credit risk. Default swaps are the most popular of these products, accounting for approximately 38% of outstanding notional balance as of December 1999, according to the BBA 1999/2000 Credit Derivatives Survey. In addition to portfolio credit default swaps, other more established products that Moody's rates include synthetic CDOs (CLOs and arbitrage CBOs).

An increasingly popular product is the credit-linked note with payment contingent upon the time and identity of the first-, second-, or *i*th-to-default entity in a basket of reference entities. According to the BBA, credit-linked notes and basket products jointly accounted for approximately 14% of the market in December 1999[1].

The market for rated *i*th-to-default basket CLNs has grown exponentially over the last two years. In 2000, Moody's rated just 4 first-to-default transactions representing $234 million of rated notes. The number of *i*th-to-default transactions increased to 25 in December 2001, accounting for $1.1 billion of rated notes.

This special report describes Moody's approach to rating *i*th-to-default credit-linked notes ("*i*TD CLNs"), focusing on the quantitative and legal issues we consider in assigning ratings to these transactions.

Moody's bases the rating of *i*TD CLNs on the concept of expected loss and employs Monte Carlo simulation, rather than the classical Binomial Expansion Technique, as the analytical tool of choice. This is due to the

1 The BBA 1999/2000 Credit Derivatives Survey of 30 institutions is contained in the document: British Bankers Association Credit Derivatives Report, 1999/2000. BBA plans to update the survey in late summer 2002.

importance of the timing of default and the identity of the defaulted entity. The rating of the reference entity, combined with default correlation assumptions applicable to the portfolio, enables Moody's to simulate the time and identity of the *i*th entity to default. Recovery rates are notoriously difficult to estimate. Moody's simulates the recovery rate of the qualifying reference obligation (typically the deliverable obligation since physical settlement is often specified) using a Beta distribution constructed to take account of the obligation's seniority in the entity's capital structure and the legal jurisdiction governing the entity's assets. Finally, Moody's links the recovery rate of the obligor to the state of the economy. This captures the fact that recovery rates tend to move with market gyrations in a particular industry and/or jurisdiction.

The *i*TD CLNs are typically issued through repackaging programmes and occasionally through the arranger's MTN programmes. We first describe Moody's approach to rating credit-linked notes issued under MTN programmes. This is important for basket credit derivatives in situations where the regulatory environment prevents investors from writing protection under credit default swaps. Investors, typically insurance companies, therefore purchase MTNs collateralised by **Aaa**-rated notes or linked to the rating of the issuer/arranger. The terms of the CLN are identical to those of the underlying credit default swap; that is, payment is contingent upon the time and identity of the *i*th reference entity to default in a given basket.

We then discuss Moody's approach to modelling correlated default and correlated recovery rates. The report concludes with a description of the legal issues that Moody's considers in rating *i*TD CLNs. The appendix provides examples of *i*TD CLNs for $i = 1,2$, and 3.

RATING METHODOLOGY FOR CREDIT-LINKED MEDIUM TERM NOTES

In the past, Moody’s assigned credit ratings to MTN programmes, with all notes issued under a programme being assigned the programme’s rating. This process was upgraded in 1996 to take into account the credit-related developments in MTN programmes, particularly with respect to credit-linked MTNs[2]. Since credit-linked notes expose investors to the risk of default by the reference entities and the issuer of the notes, Moody's incorporates the rating of the issuer in our estimation of the expected loss to the *i*TD note holder, as investors could still suffer a loss even when no credit events occur in the reference basket but the issuer defaults. This way, Moody’s arrives at a rating judgement that incorporates the risk of default by either or all of the parties supporting the transaction. Moody’s also incorporates an assessment of default correlation or diversification in the analysis. For example, a basket first-to-default CLN might reference two **Baa1** and two **Baa3** rated entities. Assuming that the four entities are highly correlated would suggest a rating of **Baa3**. On the other hand, supposing that the entities are independent would lead to the addition of expected losses, yielding a rating of **Ba3**.

WHAT ARE *I*TH-TO-DEFAULT CREDIT-LINKED NOTES?

Investors in conventional *i*th-to-default credit-linked notes gain credit exposure to a small basket of two or more reference entities (and the issuer or the collateral). These are bespoke transactions that allow investors to tailor a basket according to their specific risk appetite.

The notional amount of the credit-linked note is typically the same as the exposure to each reference entity, and the scheduled maturity of the transaction is typically the same as the tenor of the protection sought for each entity (i.e., each reference entity has obligations that mature on or after the scheduled maturity of the note, which varies between 3 and 7 years).

2 *Moody’s Special Comments*, Moody’s Announces New Medium-term Note Rating Process, January 1996; Moody’s Extends New Medium-term Note Rating Process to Euro MTN Markets, February 1997.

In the event that less than *i* credit events occur in the reference basket on or prior to the scheduled maturity date, the note redeems at par at maturity. In the event that *i* or more credit events occur on or prior to the scheduled maturity date, then assuming that physical settlement is specified (which is more common than the cash settlement alternative), the issuer delivers the notional amount of deliverable obligations of the affected reference entity in return for full par payment (equivalent to the exposure to the *i*th reference entity to suffer a credit event).

Moody's has rated variations of the plain vanilla *i*TD CLNs such as the *i*th- and *(i+1)*th-to-default CLN. Here, the notional amount of the notes is characteristically greater that the exposure to each reference entity. Upon the occurrence of the *i*th credit event in such a transaction, the protection buyer delivers to the protection seller (again assuming that physical settlement is specified) the notional amount of deliverable obligations of the affected reference entity equal to the lower of (a) the notional amount of the exposure to that reference entity, and (b) the outstanding notional amount of the note in return for full par payment of the notional amount of the *i*th reference entity. The transaction terminates when the *(i+1)*th credit event occurs, with the protection buyer delivering to the protection seller the notional amount of deliverable obligations of the affected reference entity equal to A, which is the lower of (a) the notional amount of the exposure to that reference entity, and (b) the outstanding notional amount of the note, in return for full par payment of the amount A calculated above.

HOW DOES MOODY'S EVALUATE *I*TH-TO-DEFAULT CREDIT-LINKED NOTES?

By basing the quantitative component of its ratings on the concept of expected loss, Moody's ensures that not only is the frequency of credit events in an *i*th-to-default transaction estimated, but so is the severity of loss following a credit event. This approach is beneficial to investors since, unlike an approach focusing exclusively on default frequency (the "weak link" approach) as pursued by some rating agencies, expected loss incorporates both the joint probability that *i* credit events occur in the reference basket, and the loss that investors would suffer following the credit events. A rating agency pursuing the weak link approach would assign the rating of the lowest-rated entity to a first-to-default CLN, which of course, greatly understates the credit risk taken by investors.

Moody's analysis of an *i*TD CLN begins with the determination of the promise made to the note holder by the issuer. Moody's examines several possible loss scenarios relative to the promise. Specifically, loss scenarios are generated using simulated recovery rates and simulated defaults following Moody's idealised default frequencies (adjusted for "soft" credit events). The present value of the cash flows generated in each Monte Carlo scenario is then compared with the promise to the note holder; that is, the amount that the note holder would receive in a no-credit-event scenario. A loss occurs only in scenarios where the present value of the cash flows falls short of the present value of the promised cash flows. The expected loss is the average of the losses suffered in all simulated scenarios. This is expressed as a fraction of the initial investment. Once the *i*TD CLN's expected loss is determined, the next step is to associate a letter rating to this quantity. Moody's accomplishes this task by way of a benchmarking procedure whereby we compare the *i*TD CLN's expected loss to those of conventional bullet bonds of the same maturity and assign to the note the rating that matches the CLN's expected loss most closely[3]. Moody's Monte Carlo *i*th-to-default model is available to market participants for a fee.

3 Moody's Special Report, Moody's Refines its Approach to Rating Structured Notes, July 1997

Default Modelling

Defaults are simulated in annual intervals using the marginal default rates embedded in Moody's idealised cumulative default probabilities (Table 1).

Table 1

Idealized Cumulative Default Rates

	Year									
Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0.00005%	0.00020%	0.00070%	0.0018%	0.0029%	0.0040%	0.0052%	0.0066%	0.0082%	0.0100%
Aa1	0.0006%	0.0030%	0.0100%	0.0210%	0.0310%	0.0420%	0.0540%	0.0670%	0.0820%	0.1000%
Aa2	0.0014%	0.0080%	0.0260%	0.0470%	0.0680%	0.0890%	0.1110%	0.1350%	0.1640%	0.2000%
Aa3	0.0030%	0.0190%	0.0590%	0.1010%	0.1420%	0.1830%	0.2270%	0.2720%	0.3270%	0.4000%
A1	0.0058%	0.0370%	0.1170%	0.1890%	0.2610%	0.3300%	0.4060%	0.4800%	0.5730%	0.7000%
A2	0.0109%	0.0700%	0.2220%	0.3450%	0.4670%	0.5830%	0.7100%	0.8290%	0.9820%	1.2000%
A3	0.0389%	0.1500%	0.3600%	0.5400%	0.7300%	0.9100%	1.1100%	1.3000%	1.5200%	1.8000%
Baa1	0.0900%	0.2800%	0.5600%	0.8300%	1.1000%	1.3700%	1.6700%	1.9700%	2.2700%	2.6000%
Baa2	0.1700%	0.4700%	0.8300%	1.2000%	1.5800%	1.9700%	2.4100%	2.8500%	3.2400%	3.6000%
Baa3	0.4200%	1.0500%	1.7100%	2.3800%	3.0500%	3.7000%	4.3300%	4.9700%	5.5700%	6.1000%
Ba1	0.8700%	2.0200%	3.1300%	4.2000%	5.2800%	6.2500%	7.0600%	7.8900%	8.6900%	9.4000%
Ba2	1.5600%	3.4700%	5.1800%	6.8000%	8.4100%	9.7700%	10.7000%	11.6600%	12.6500%	13.5000%
Ba3	2.8100%	5.5100%	7.8700%	9.7900%	11.8600%	13.4900%	14.6200%	15.7100%	16.7100%	17.6600%
B1	4.6800%	8.3800%	11.5800%	13.8500%	16.1200%	17.8900%	19.1300%	20.2300%	21.2400%	22.2000%
B2	7.1600%	11.6700%	15.5500%	18.1300%	20.7100%	22.6500%	24.0100%	25.1500%	26.2200%	27.2000%
B3	11.6200%	16.6100%	21.0300%	24.0400%	27.0500%	29.2000%	31.0000%	32.5800%	33.7800%	34.9000%
Caa	26.0000%	32.5000%	39.0000%	43.8800%	48.7500%	52.0000%	55.2500%	58.5000%	61.7500%	65.0000%

The annual marginal default rate gives the default probability of a reference entity in a given year, conditional upon the entity not having defaulted in a previous year. Thus, for a **Baa2**-rated entity, the probability that the entity defaults in year 3 is given by the probability that the entity defaults in or prior to year 3 (0.83%) minus the probability that the entity defaults in or prior to year 2 (0.47%) divided by the probability that the entity survives in year 1 and year 2 (99.53%). Hence the marginal default rate in year 3 for the **Baa2**-rated entity is 0.3617%.

The credit quality of reference entity *i* in year t is modelled as a standard normal random variable $Z_{i,t}$. The credit quality of the issuer of the CLN or of the collateral is also modelled in this way. This component of our model is similar to structural-form models following the Merton approach[4] in that it essentially models the returns of the firm or its asset value. Unlike in a pure structural-form setting, our model uses as an input, the rating of the reference entity to obtain the threshold value of the firm below which a credit event occurs. The entity's default threshold in year t, $\alpha_{i,t}$, is computed by applying the inverse of the cumulative univariate normal distribution function Φ^{-1} to the marginal default probability $m_{i,t}$ of the reference entity in year t; that is, $\alpha_{i,t} = \Phi^{-1}(m_{i,t})$.

In a model that assumes independence of the reference entities, entity *i* suffers a credit event in year t if $Z_{i,t} < \alpha_{i,t}$. Although the assumption of independence is conservative for a first-to-default CLN, it is both inaccurate and understates default probabilities for *i*TDs when $i \geq 2$. The random variable $Z_{i,t}$ is generated such that it is a blended random variable taking account of reference entities' asset correlation within an industry and within a geographical region[5]. Moody's two-factor model is more precisely described as follows:

$$Z_{i,t} = \lambda_{i,1} Z_{R,t} + \lambda_{i,2} Z_{I,t} + \lambda_{i,3} Z_{F,t}, \text{ such that } \sum_{n=1}^{3} \lambda_{i,n}^2 = 1, \quad \text{..........................(1)}$$

4 Merton, Robert C., On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, Journal of Finance, 2, 449-471, 1974.

5 The model allows for user-defined categorisations into industry and geographical regions. Such a categorisation relies on data sufficiency to estimate the intra-industrial and intra-regional correlation coefficients.

where the Z's on the right-hand side of equation (1) are independent, standard normal random variables representing respectively, the economic performance of a reference entity's region and industry (in year t), while the third factor represents firm-specific contributors to default such as fraud, event risk or over-leverage. The λ's in the equation represent the respective weights or contributions to an entity's default by industry, region, and the entity itself. They are linked to default correlation coefficients as shown in equation (2) below. The model is two-factor because correlation between a pair of entities results from the entities operating in the same industry and/or legal jurisdiction. The third component on the right-hand side of equation (1) is firm-specific and therefore idiosyncratic.

The importance of the requirements that the squares of the coefficients in equation (1) sum to one, and that the Z's be independent can be demonstrated by calculating the autocovariance of the random variable $Z_{i,t}$:

$$\begin{aligned} Cov(Z_{i,t}, Z_{i,t}) &= \lambda_{i,1}^2 Cov(Z_{R,t}, Z_{R,t}) + \lambda_{i,2}^2 Cov(Z_{I,t}, Z_{I,t}) + \lambda_{i,3}^2 Cov(Z_{F,t}, Z_{F,t}) \\ &+ 2\{\lambda_{i,1}\lambda_{i,2} Cov(Z_{R,t}, Z_{I,t}) + \lambda_{i,2}\lambda_{i,3} Cov(Z_{I,t}, Z_{F,t}) + \lambda_{i,1}\lambda_{i,3} Cov(Z_{R,t}, Z_{F,t})\} \\ &= \lambda_{i,1}^2 + \lambda_{i,2}^2 + \lambda_{i,3}^2, \end{aligned}$$

which is equal to one by definition (since each entity is perfectly correlated with itself). In the absence of the above assumptions, $Cov(Z_{R,t}, Z_{I,t}) \neq 0,$ for example. For two reference entities i and j in the same industry and geographical region, the correlation at time t is given by

$$Corr(Z_{i,t}, Z_{j,t}) = \frac{Cov(Z_{i,t}, Z_{j,t})}{\sqrt{Var(Z_{i,t})Var(Z_{j,t})}} = Cov(Z_{i,t}, Z_{j,t}) = \lambda_{i,1}\lambda_{j,1} + \lambda_{i,2}\lambda_{j,2} \quad \text{..........................(2)}$$

The variances of the random variables in the denominator of equation (2) are each unity since they are standard normal random variables. If the reference entities in equation (2) were in the same geographical region but different industries, the correlation would be $\lambda_{i,1}\lambda_{j,1}$, whereas if they were in the same industry but different geographical regions, the correlation would be $\lambda_{i,2}\lambda_{j,2}$.

Estimating the $\lambda's$

The $\lambda's$ can be estimated through the use of a copula function and historical data to obtain the correlation coefficient between entity i and entity j. A copula function provides a connection between the marginal default probability distributions of the reference entities, and their multivariate distribution. Estimating the $\lambda's$ in equation (2) for all pairs of reference entities can rapidly become intractable even for baskets of 5-10 entities (as is *de rigueur* in *i*TD transactions). Assuming that each entity's default is similarly influenced by the prevailing economic environment and by its own firm-specific misfortune, then $\lambda_{i,n} = \lambda_n$ for all reference entities i , and for $n = 1,2,3.$ The assumption implies, for example, that the contribution from the industry to an entity's default is the same across different industries. This is an intuitive assumption since no single, well defined industry category can be said to expose companies to default risk more than all others in the life of the CLN. Equation (1) therefore becomes

$$Z_{i,t} = \lambda_1 Z_{R,t} + \lambda_2 Z_{I,t} + \lambda_3 Z_{F,t}, \quad \sum_{n=1}^{3} \lambda_n^2 = 1. \quad \text{..........................(3)}$$

Thus, using the bivariate normal copula function (the integrand in the double integral below), the correlation between entity i and entity j, ρ_{ij}, is the only unknown quantity in the double integral (which is also the joint probability of default of entities *i* and *j*):

$$\Pr(Z_{i,t} < \alpha_{i,t}, Z_{j,t} < \alpha_{j,t}) = \int_{-\infty}^{\alpha_{i,t}} \int_{-\infty}^{\alpha_{j,t}} \phi(x, y, \rho_{ij})\, dxdy,$$

where $\phi(x, y, \rho_{ij})$ is the bivariate normal density function with correlation coefficient ρ_{ij}.

Once the correlation coefficients are obtained in this way, the λ's can be found by the use of Cholesky decomposition, which is equivalent to saying that for two entities *i* and *j* in the same industry and legal jurisdiction, the correlation ρ_{ij} is given by $\rho_{ij} = \lambda_1^2 + \lambda_2^2 = 1 - \lambda_3^2$. The process can be repeated for entities in the same industry but different geographical regions to obtain λ_2, and the remaining coefficient, λ_1, will follow from the fact that the squares of the coefficients sum to unity.

Moody's final correlation assumptions are determined on a case by case basis and will depend on the industries and geographical regions of the reference entities.

Recovery Modelling

Moody's believes that the use of fixed recovery rates in credit risk modelling can greatly underestimate severity of loss. Although the link between severity of loss and the state of the economy is well-known (see, for example, Moody's Special Comments[6]), almost all credit risk management models and tools treat severity of loss as independent of expected default rates, preferring instead, to treat recovery rates either as a fixed quantity or as a function of historic average recovery rates and seniority[7]. Moody's constructs a recovery rate distribution for each reference entity based on the entity's legal jurisdiction and the seniority of the qualifying reference obligation in the capital structure of the entity. In addition, we correlate recovery rates with the state of the economy by using the results of the default simulation module[8]. Moody's uses a Beta distribution because of its natural properties of boundedness at zero and 100% (the domain of a general Beta distribution function is not necessarily the unit interval).

Recall that the default module presented above allows for the simulation of the state of the economy at time t in the form of random variables $Z_{R,t}$ and $Z_{I,t}$ in equations (1) and (3). For recovery modelling, Moody's generates a blended standard normal random variable as in equation (3), adjusting two features: the weights and the obligor-specific random variable (since firm-specific factors affecting an entity's default likelihood are not necessarily those driving recovery values). We thus have equation (4) below:

$$Z_{i,t}^r = \eta_1 Z_{R,t} + \eta_2 Z_{I,t} + \eta_3 Z_{F,t}^r, \quad \text{with } \sum_{n=1}^{3} \eta_n^2 = 1, \quad \text{..........................(4)}$$

where the super-script r represents random variables associated with recovery rate simulation only, as distinct from those associated with both recovery rate and default simulations. As with default simulation, the weights η in equation (4) are determined on a case by case basis.

6 *Moody's Special Comments*, Debt Recoveries for Corporate Bankruptcies, June 1999; Default and Recovery Rates of Corporate Bond Issuers, February 2002.

7 Recoveries are stochastic in credit value at risk models such as J.P. Morgan's CreditMetrics©, McKinsey's CreditPortfolioView©, and KMV's Portfolio Manager©, and generally follow a Beta distribution; they are fixed in CSFP's CreditRisk+©.

8 This approach was first suggested in a one-factor model by Jon Frye in the article: Collateral Damage, *Risk*, April 2000. Two further articles by the same author provide empirical evidence for the formulation: Collateral Damage Detected, Federal Reserve Bank of Chicago, Working Paper, Emerging Issues Series, October, 1-14; and Depressing Recoveries, *Risk,* November 2000.

Observe that the correlation between an entity's default and recovery rates at time t is given by

$$Corr(Z_{i,t}, Z^r_{i,t}) = Cov(Z_{i,t}, Z^r_{i,t}) = \lambda_1\eta_1 + \lambda_2\eta_2.$$

The dependence on time and obligor drop out because of our assumption that correlation in defaults and recoveries are driven by non-temporal and macro-economic factors. This time independence does not necessarily lead to loss of generality in the model since the availability of sufficient data could still support calibration of the model such that defaults and recoveries are simulated based on regimes or credit cycles; that is, the λ's and $\eta's$ in equations (3) and (4) could be made functions of time without altering the basic structure of the model.

Using the cumulative normal distribution function Φ, we obtain the probability that a given recovery rate is observed from a Beta distribution with parameters *a* and *b*, and probability density function f given by:

$$f(x) = \frac{x^{a-1}(1-x)^{b-1}}{\beta(a,b)}, \ 0 \le x \le 1,$$

where

$$\beta(a,b) = \int_0^1 y^{a-1}(1-y)^{b-1}dy, \qquad a,b > 0.$$

The probability of drawing a recovery rate θ from a Beta distribution with parameters a and b (calculated from the mean and standard deviation of recovery rates for a given seniority and a given jurisdiction[9]) is thus $\Phi(Z^r_{i,t})$, so that $\theta = F^{-1}(\Phi(Z^r_{i,t}), a, b)$, F^{-1} being the inverse of the cumulative Beta distribution function.

LEGAL ISSUES

As earlier described, *i*TD CLNs are typically issued through repackaging programmes and occasionally through the arranger's MTN programme. In addition to the quantitative issues outlined above, Moody's considers legal and documentation risks inherent in the transactions. The objective is to ensure that the programmes have only the liabilities associated with the contemplated transactions. To achieve this goal, Moody's reviews the repackaging programme's formative and governing documentation, including limits on the ability to incur debt and the covenants of various parties who could initiate bankruptcy proceedings against the programme. The special purpose vehicles should be subject, for example, to the standard requirement of limited recourse to the assets of the issuer.

The legal analysis of *i*TD CLNs issued out of an SPV under a repackaging programme is not unlike that of CLNs issued out of MTN programmes. As previously described, the note holder in an MTN programme is exposed to the risk of credit events affecting the reference entities and the issuer of the notes. This also applies in the case where the *i*TD CLN is issued out of a repackaging programme. The SPV issues the *i*TD CLN and delivers the proceeds to the arranger as the swap counterparty. The arranger pays the coupon on the notes. If the arranger defaults, the transaction unwinds; note holders are therefore also exposed to the default risk of the arranger.

9 Given the mean μ and variance σ^2 of recovery rates from historical data, the Beta parameters can be estimated using the transformations $a = \mu^2(1-\mu)/\sigma^2 - \mu$ and $b = (1-\mu)(\mu(1-\mu)/\sigma^2 - 1)$.

Evaluation of the credit risk in the documentation hinges on the definition of credit events, since these events trigger losses to *i*TD CLN investors. Moody's reviews the documentation to ensure that the definition of credit events is unambiguous, and that it constitutes a risk generally associated with credit risk, where such risk could also be clearly determined by third parties. The 1999 ISDA Credit Derivatives Definitions (along with the various supplements) is the documentation standard utilised[10].

10 Moody's concerns on the use of ISDA documentation are outlined in the Special Report: *Understanding The Risks in Credit Default Swaps*, March 2001. Following the issues raised in this report, ISDA recently excluded Obligation Acceleration, Obligation Default and Repudiation/Moratorium from the menu of credit events.

APPENDIX: EXAMPLES OF *I*TH-TO-DEFAULT TRANSACTIONS

We provide an example of a first-, a second- and a third-to-default CLN transaction each referencing the same basket in order to demonstrate the methodology outlined in this special report. It is assumed that the issuer collateralises its obligations under the CLNs with **Aaa**-rated collateral acceptable to Moody's. It is further assumed that the notional amount of each CLN is exactly the same as the notional amount of each reference entity in the basket (Euro 50 million, although this amount is extraneous in itself). A stress factor of 20% is applied to the marginal default probability of each reference entity in each year to account for any "soft" credit events in the documentation.

The scheduled maturity date of each transaction is in 5 years. The first-to-default CLN promises to pay 1-year Euribor (which we assume to be flat at 3.90% per annum over the 5 years) plus a spread of 1.50% per annum. The second- and third-to-default CLNs promise to pay 1-year Euribor plus 0.75% per annum and 0.45% per annum, respectively.

REFERENCE ENTITIES

					Recovery Parameters		Beta Distn Parameters	
Identity	**Senior Unsecured Rating**	**Seniority of Deliverable Obligation**	**Moody's Industry Category**	**Country of Domicile**	**Mean**	**StDev**	***a***	***b***
Entity 1	**Aa1**	Senior Unsecured	Automobile	USA	50%	30%	0.89	0.89
Entity 2	**Aa2**	Senior Unsecured	Automobile	USA	50%	30%	0.89	0.89
Entity 3	**Aa3**	Senior Unsecured	Banking	USA	50%	30%	0.89	0.89
Entity 4	**A1**	Subordinated	Banking	USA	35%	20%	1.64	3.05
Entity 5	**A2**	Senior Unsecured	Banking	UK	20%	10%	3.00	12.00
Entity 6	**A1**	Senior Unsecured	Banking	UK	20%	10%	3.00	12.00
Entity 7	**Aa3**	Senior Unsecured	Finance	UK	20%	10%	3.00	12.00
Entity 8	**Aa2**	Senior Unsecured	Healthcare	UK	20%	10%	3.00	12.00
Entity 9	**Aa1**	Senior Unsecured	Oil and Gas	Netherlands	20%	10%	3.00	12.00
Entity 10	**Aa2**	Senior Unsecured	Electronics	Netherlands	20%	10%	3.00	12.00

In this example, we have set intra-industrial and intra-regional correlation at 15% each, with respect to both recovery and default simulations. Thus two entities in the same region and industry are assumed to have a default correlation of 30% and a recovery correlation of 30%. Correlation between default and recovery rates is also 30%. The default correlation assumption is similar to Moody's assumptions in calculation Diversity Scores in CDO analysis, where depending on the diversification cap of contributions from a given industry, say 3 to 5, the implied correlation can range from 20% to 33%[11]. This implies that $\lambda_1 = \lambda_2 = \eta_1 = \eta_2 = 0.3873$, and $\lambda_3 = \eta_3 = 0.8367$ in equations (3) and (4). The recovery rates in the above table are presented for the purpose of illustration only. Tables 2, 3 and 4 provide Moody's Monte Carlo model statistics for the respective CLNs. *Table 5* shows the effect of increasing concentration (equivalent to increasing the correlation coefficients) in a given industry and/or legal jurisdiction on the expected loss of the second-to-default CLN.

11 The implied correlation can be computed from the formula $D = n/(1+(n-1)\rho)$ by fixing *D* (at 4, say) and letting *n* be very large (giving $\rho = 25\%$ in this example). This formula is derived, for example, in Appendix II of the Special Report: Moody's Approach to Rating Multisector CDOs, September 2000.

Table 2

Monte Carlo Statistics	First-to-Default
Expected Loss (EL)	0.962848%
Standard Deviation	7.81718%
Standard Error (SE)*	0.01563%
EL plus SE	**0.978482%**
Rating	**Baa2**
EL Mid-points for comparison	
Baa1	0.60500%
Baa2	0.86900%
Baa3	1.67750%

*This helps set the number of Monte Carlo simulations (250,000 in this case) since the standard error is given by $\sigma / \sqrt{n}$, where σ is the standard deviation of the resulting loss distribution, and n is the number of simulations.

Table 3

Monte Carlo Statistics	Second-to-Default
Expected Loss	0.014612%
Standard Deviation	0.97015%
Standard Error	0.00194%
EL plus SE	**0.016552%**
Rating	**Aa1**
Mid-points	
Aaa	0.00160%
Aa1	0.01705%
Aa2	0.03740%

Table 4

Monte Carlo Statistics	Third-to-Default
Expected Loss	0.001284%
Standard Deviation	0.31181%
Standard Error	0.00062%
EL plus SE	**0.00191%**
Rating	**Aaa**
Mid-points	
Aaa	0.00160%
Aa1	0.01705%

Table 5: EL of Second-to-Default versus Increasing Concentration

Default Correlation		Recovery Correlation		
Regional	**Industrial**	**Regional**	**Industrial**	**EL**
0%	0%	0%	0%	0.00752%
0%	5%	15%	15%	0.00883%
5%	0%	15%	15%	0.00968%
5%	5%	15%	15%	0.01023%
5%	10%	15%	15%	0.01159%
10%	10%	15%	15%	0.01339%
15%	15%	15%	15%	0.01655%
15%	15%	15%	20%	0.01836%
15%	15%	20%	20%	0.01925%
20%	15%	15%	15%	0.02144%
20%	20%	15%	15%	0.02541%
20%	20%	20%	20%	0.02781%
20%	25%	20%	25%	0.03088%

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